# Topics in mathematical biology (L8)

***Non-Examinable (Graduate Level)***

**Dr Jonathan Swinton and Professor Julia Gog**

This course will consist of two independent series of 4 lectures each on mathematical phyllotaxis and mathematical epidemiology.

### Epidemiology for pandemics

This course will begin with a very rapid review of the ordinary differential equation '$SIR$' model for the spread of epidemics, often taught in undergraduate mathematics courses. This model allows a simple demonstration of the close theoretical relationship between the basic reproductive number, $R_0$, usually defined through the linear stability of the disease-free state, and the critical population fraction to vaccinate to achieve herd immunity, $1 - 1/R_0$. The relationship is unusual in mathematical biology in having been empirically validated over many decades and having been relied on in many applications to significant human and animal disease. As a consequence, there have been many generalisations into, for example stochastic or spatially extended models, forming a substantial field of modelling-based mathematical epidemiology, as distinct from more statistical approaches.

This course will discuss a number of different UK policy actions which have drawn on mathematical epidemiology, including badger culls, measles vaccination, and of course covid lockdowns. Rather than ask when models have been right, we will more fruitfully explore what mathematical techniques and what mathematician's skills have and perhaps have not been useful. Dr Swinton will give three lectures on past and present epidemic models, while Professor Gog will give a lecture on her current experience of being a mathematical epidemiologist in a policy world.

### Mathematical Phyllotaxis

Fibonacci numbers in plants, such as in sunflower spiral counts, have long fascinated mathematicians. Most analyses are variants of a Standard Model in which organs are treated as point nodes successively placed on a cylinder according to a given function of the previous node positions, not too close or too far away from the existing nodes. These models usually lead to lattice solutions. As a parameter of the model, like the diameter of the cylinder, is changed, the lattice can transition to another, more complex lattice, with a different spiral count. It can typically be proved that these transitions move lattice counts to higher Fibonacci numbers. While mathematically compelling, empirical validation of the Standard Model is as yet weak.

The course by Dr Jonathan Swinton will begin with a gallery of examples of Fibonacci patterning and a survey of the quantitative datasets available. We will give a brief history of mathematical approaches, including a partially successful attempt by Alan Turing. We study the mathematics of lattices on cylinders and classify lattice space using a fractal decomposition with close links to number theory. We will see the general properties a model will need to have to lead generically to Fibonacci structure. We will then introduce a range of biological models and survey the links between models and data, from the statistical to the molecular.

This non-examinable course will consists of four lectures and an examples class. It is strongly recommended for anyone who intends to offer the Mathematical Phyllotaxis essay.

### Prerequisites

There are no pre-requisites for this course.

The basic mathematics of the $SIR$ model is treated in most mathematical biology texts; details of the papers that will be revieiwed will be circulated prior to the lectures. Some previous experience with bifurcation theory may make the central idea of the Fibonacci theory more attractive.

### Background reading: epidemiology

- O Diekmann and JAP Heesterbeek, *Mathematical Epidemiology of Infectious Diseases*, Wiley, 2000

### Background reading: phyllotaxis

- R Jean, *Phyllotaxis a Systemic Study*, CUP 1994.
- R Jean and D Barabé, *Symmetry in Plants*, World Scientific, 1998.
- Course text: J Swinton, *A Textbook of Mathematical Phyllotaxis*, Infang Publishing, to be published 2021

The course text will be available online to Part III students from 1 October.

### Additional support

There will be one examples class where examples from the course text will be discussed. Office hours will not be provided, but the lecturer can be contacted directly about the course material.