Model Theory and Non-Classical Logic (M24)

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This course builds upon the foundations laid out by the Part II course 'Logic and Set Theory', introducing some important concepts and tools in modern Logic. It is broadly divided into a chapter dedicated to Model Theory, and another to Intuitionistic Logic.

There are two prevailing analogies that aim to explain Model Theory. The first says that 'model theory = universal algebra + logic', viewing it as a generalisation of algebra that allows for relations instead of just operations. The other sees it instead as geometrically flavoured, with logical formulae playing a role akin to equations in defining your structure of interest: 'model theory = algebraic geometry − fields'. Regardless of the point of view, the techniques we will discuss will give us a finer control over the structures described by a first-order theory.

Intuitionistic logic departs from classical logic by adopting a new interpretation for the logical connectives and quantifiers that makes it constructive. We will discuss its syntax and semantics (which requires new notions of model with accompanying completeness theorems), and see how it compares to its classical counterpart.

We will cover most or all of the following topics:

- Definable sets, elementary substructures, and the Tarski-Vaught test;
- Universal theories;
- Skolemisation and the Löwenheim-Skolem Theorem;
- Quantifier elimination;
- Ultraproducts and Łoś's Theorem;
- Nonstandard models of arithmetic;
- The Omitting Types Theorem;
- The Ehrenfeucht–Mostowski Theorem;
- Natural deduction and intuitionistic logic;
- The simply typed λ-calculus and the Curry-Howard correspondence;
- Heyting and Kripke semantics for intuitionistic logic;
- Negative translation;
- Recursive models and Tennenbaum's Theorem.

The course will primarily be of interest to students in Foundations, Combinatorics, and Algebraic Geometry.

Prerequisites

Familiarity with the contents of the Part II course *Logic and Set Theory* (or equivalent) is essential. The course will assume acquaintance with first-order logic, including its syntax and basic results (e.g., completeness, compactness), Zorn's Lemma, ordinals, and cardinals.

Knowledge of basic computability theory (recursive and recursively enumerable sets, the Halting Problem, the s-m-n theorem) to the level of the Part II course *Automata & Formal Languages* (or equivalent) is desirable, but not essential. The relevant facts and definitions will be revised at a fast pace if needed.

Literature

Students are likely to find the following useful, particularly references 2, 3, and 8:

1. C.C. Chang, H.J. Keisler, *Model Theory*, Studies in Logic and the Foundations of Mathematics, Elsevier, Volume 73, 1990.
2. W. Hodges. *Model theory*, Encyclopedia of Mathematics and its Applications, Cambridge University Press, 1993.
3. D. Marker. *Model Theory: An Introduction*, Graduate Texts in Mathematics, Springer, New York, 2002.
4. J. L. Bell, A. B. Slomson. *Models and Ultraproducts: An Introduction*, Dover Books on Mathematics Series, Dover Publications, 2013.
5. A. S. Troelstra, D. van Dalen. *Constructivism in Mathematics: an introduction*, Vol. 1, Elsevier, 1988.
6. J. Girard, Y. Lafont, P. Taylor. *Proofs and Types*, Cambridge University Press, 1989.
7. R. Kaye. *Models Of Peano Arithmetic*, Oxford Logic Guides **15**, Oxford University Press, 1991.
8. D. van Dalen. *Logic and Structure.* Fourth edition. Springer, 1997.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.