Mapping class groups (M16)

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Let S be a compact, orientable surface, with finitely many punctures and boundary components. The group of orientation-preserving self-homeomorphisms of S, $\mathrm{Homeo}^+(S)$, is too large to conveniently study, so we instead pass to the quotient

$$\mathrm{Mod}(S) = \mathrm{Homeo}^+(S)/\mathrm{Homeo}_0(S)$$

by factoring out the path component of the identity element. The resulting group – the *mapping class group* of S – is both tractable to study, and encodes a great deal of information about the topology and geometry of S. Mapping class groups are ubiquitous, appearing in subjects as diverse as algebraic geometry, combinatorial group theory, symplectic geometry, dynamics and 3-manifold topology.

This course introduces some of the basic techniques used to study mapping class groups, and applies them to compute examples and to prove some fundamental results. We will cover the following topics.

(a) The bigon criterion, which makes it possible to determine if two elements of $\mathrm{Mod}(S)$ are equal.

(b) The simplest examples of mapping class groups, including the important case when S is a torus.

(c) Dehn twists, the most fundamental mapping classes of infinite order.

(d) The complex of curves, which will enable us to prove that $\mathrm{Mod}(S)$ is finitely generated.

Pre-requisites

Part Ib Geometry is essential. Part II Algebraic Topology is essential. Part II Riemann Surfaces is useful. Part III Algebraic Topology, taken concurrently, is useful.

Literature

1. B. Farb and D. Margalit *A primer on mapping class groups.* Princeton Mathematical Series, 49. Princeton University Press, Princeton, NJ, 2012. xiv+472 pp.

Additional support

Printed notes are available online. Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.