Knots (L24)

Jacob Rasmussen

This course will give an introduction to the theory of knots and its relation to other areas of low-dimensional topology. A knot in S^3 is a smooth embedding of the circle into the 3-sphere. Two such embeddings are equivalent if they can be continuously deformed into each other while remaining in the class of knots. Although at first glance this notion may seem incredibly specialized, knot theory plays an important role in the study of 3 and 4-dimensional manifolds, and many results in the theory of knots are best understood in this context.

The main emphasis of the course will be on geometric constructions, geometric problems, and the tools used to solve them. I hope to cover the following topics:

- **Knot Invariants:** The Jones polynomial, the crossing number, and alternating knots. The fundamental group and the Alexander polynomial. Fox calculus.
- **3-dimensional topology:** Fibred knots and Seifert genus. The multivariable Alexander polynomial and Thurston norm. The branched double cover and Dehn surgery. The unknotting number.
- **Tangles:** Prime decomposition. Rational tangles and rational knots. The Jones polynomial of a tangle. Mutation.
- **4-dimensional topology:** Slice genus and the signature. The concordance group. Khovanov homology and strongly quasipositive knots.

Prerequisites

Algebraic Topology. I will assume familiarity with the fundamental group, homology, and cohomology.

Literature

Lickorish's book is a good general introduction. Rolfsen's book is a bit dated, but valuable for its coverage of the Alexander polynomial.

1. W. B. R. Lickorish *An Introduction to Knot Theory* GTM 175, Springer Verlag, 1997.
2. D. Rolfsen *Knots and Links*, AMS Chelsea publishing (reprint), 1976.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.