Differential Geometry (M24)

Jack Smith

Differential geometry is the study of manifolds—spaces built from smoothly gluing together open sets in Euclidean space—and structures that live on or in them. The goal of this course is to introduce the main ideas on both the abstract conceptual ('coordinate-free') level and the concrete computational ('in coordinates') level, and to develop fluency in passing between them. This will lay the foundation for future study in geometry and topology, and provide the language for modern theoretical physics. Throughout the emphasis will be on building up geometric intuition. Topics will include:

- Manifolds, smooth maps, and tangent spaces. Submanifolds and transversality.
- Vector bundles and operations on them. Tangent bundle, cotangent bundle, and tensors. Vector fields, flows, and Lie derivatives.
- Differential forms, the exterior derivative, and de Rham cohomology. Integration and Stokes's theorem. Foliations and Frobenius integrability.
- Lie groups and Lie algebras. The exponential map. Lie group actions.
- Principal bundles, reduction of structure group, and associated vector bundles. Connections and curvature from multiple perspectives.
- Riemannian metrics, the Levi-Civita connection, and geodesics. The Riemann tensor and its symmetries and contractions. The Laplace–Beltrami operator and statement of the Hodge decomposition.

Prerequisites

Familiarity with point set topology (including compactness), multi-variable calculus (including the inverse function theorem), and linear algebra (including dual spaces and bilinear forms) is essential. No previous exposure to geometry will be assumed.

Literature

1. Liviu I. Nicolaescu, *Lectures on the geometry of manifolds.* 2nd edition. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2007.
2. John M. Lee, *Introduction to smooth manifolds.* 2nd edition. Graduate Texts in Mathematics, 218. Springer, New York, 2013.
3. (For a physics perspective) Mikio Nakahara, *Geometry, topology and physics.* 2nd edition. Graduate Student Series in Physics. Institute of Physics, Bristol, 2003.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.