

Cubulating spaces and groups (L16)

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This course will be an introduction to the study of non-positively curved cube complexes, which are cell complexes built out of cubes of various dimensions in such a way that they satisfy a certain combinatorial non-positive curvature condition. These objects, first introduced by Gromov as a source of examples, have been extensively utilised and developed in the last couple of decades, and can be used to tackle problems in geometric group theory, low-dimensional topology, and even robotics.

While there is no overlap in the syllabus, the material that will be covered in this course is closely related to the material that will be covered in Part III Geometric Group Theory. The students interested in either course would therefore benefit from taking both.

The course will cover a selection of topics including:

- Non-positively curved cube complexes
- Wallspaces and their dual cube complexes
- Special cube complexes, separability, and right-angled Artin groups
- Obstructions to cubulability
- Bestvina-Brady Morse theory

Prerequisites

Part IA Groups and Part II Algebraic Topology are essential; Part IB Geometry is desirable.

Literature

1. Mladen Bestvina, Michah Sageev, and Karen Vogtmann, editors. *Geometric group theory*, volume 21 of *IAS/Park City Mathematics Series*. American Mathematical Society, Providence, RI; Institute for Advanced Study (IAS), Princeton, NJ, 2014. Including lecture notes from the Graduate Summer School held at the Park City Mathematics Institute (PCMI), Park City, UT, July 1–21, 2012.
2. Indira Chatterji and Graham Niblo. From wall spaces to CAT(0) cube complexes. *Internat. J. Algebra Comput.*, 15(5-6):875–885, 2005.
3. Daniel T. Wise. *The structure of groups with a quasiconvex hierarchy*. *Annals of Mathematics Studies*, to appear.
4. Daniel T. Wise. *From riches to raags: 3-manifolds, right-angled Artin groups, and cubical geometry*, volume 117 of *CBMS Regional Conference Series in Mathematics*. Published for the Conference Board of the Mathematical Sciences, Washington, DC, 2012.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.