Direct and Inverse Scattering of Waves (L16)

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The study of wave scattering is concerned with how the propagation of waves is affected by objects, and has a variety of applications in many fields, from environmental science to seismology, medicine, telecommunications, materials science, military applications, and many others. If we know the nature of the objects and we want to find how an incident wave is scattered, we call this a 'direct scattering problem' and practical applications will include for example underwater sound propagation, light transmission through the atmosphere, or the effect of noise in built-up areas. If we measure and know the scattered field produced by an incident wave, but we do not know the nature of the objects that have scattered it, we call this an 'inverse scattering problem' and applications will include for example non-destructive testing of materials, remote sensing with radar or lidar, or medical imaging.

This course will provide the basic theory of wave propagation and scattering and an overview of the main mathematical methods and approximations, with particular emphasis on inhomogeneous and random media, and on the regularisation of inverse scattering problems. It is intended as a kind of bridge between the Applied and Computational Analysis courses and the Continuum Mechanics courses and should appeal to students interested in waves as well as in inverse problems.

The course will cover a selection of topics including:

- Boundary value problems and the integral form of the wave equation.
- The parabolic equation and Born and Rytov approximations for the scattering problem.
- Scattering by randomly rough surfaces and propagation in inhomogeneous media.
- Ill-posedness of the inverse scattering problem, and the Moore-Penrose generalised inverse.
- Regularisation methods and methods for solving some inverse scattering problems.
- Time reversal and focusing in inhomogeneous media.

Prerequisites

This course assumes basic knowledge of PDEs, and of Fourier transforms. Some familiarity with linear algebra and with basic concepts in functional analysis is helpful, though by no means necessary.

The Part III course Inverse Problems is a useful complement to part of this course, although not a prerequisite.

Preliminary Reading

These can provide some useful background:

1. C.W. Groetsch *Inverse Problems in the Mathematical Sciences.* Braunschweig 1993
2. L.D. Landau and E.M. Lifschitz *Fluid Dynamics.* Pergamon 1987 [Chapter 8]. Also available at

   users-phys.au.dk/srf/hydro/Landau+Lifschitz.pdf

Literature

1. D. Colton and R. Kress *Inverse Acoustic and Electromagnetic Scattering Theory.* Springer, 1992.
2. D.G. Crighton et al, *Modern Methods in Analytical Acoustics.* Springer, 1992.
3. H.W. Engl, M. Hanke and A. Neubauer, *Regularization of inverse problems.* Kluwer, 2000.
4. A. Ishimaru, *Wave Propagation and Scattering in Random Media.* Academic Press, 1978.
5. A. Kirsch, *An introduction to the mathematical theory of inverse problems.* Springer, 1996.
6. B. Uscinski, *The elements of wave propagation in random media.* McGraw-Hill, 1977.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a two-hour revision class in the Easter Term.