Extremal Graph Theory (E12)

Non-Examinable (Graduate Level)

Dr O. Janzer

Extremal Graph Theory studies questions of the following kind: at most how many edges can a graph on n vertices have if it satisfies a certain property? One of the most important instances of this problem is the question of how many edges an n-vertex graph can have without containing some given graph H as a subgraph (called the *extremal number* of H). Turán's theorem answers this question exactly for complete graphs, and the Erdős–Stone theorem determines the asymptotics whenever H is not bipartite. The bipartite case remains open in general, but there are many important and interesting bipartite graphs H for which the above question has been answered.

The course will cover a selection of topics including:

- The Erdős–Stone theorem
- Stability and supersaturation
- Extremal number of complete bipartite graphs
- The Erdős–Simonovits regularization lemma
- Extremal number of cycles
- Dependent random choice and Füredi's theorem on bipartite graphs with bounded degree on one side
- Extremal number of subdivisions of complete graphs and a connection to Ramsey numbers

Prerequisites

Familiarity with basic concepts and techniques in Graph Theory will be assumed.

Literature

1. B. Bollobás, *Extremal Graph Theory.* Dover Books on Mathematics.
2. Z. Füredi and M. Simonovits, *The history of degenerate (bipartite) extremal graph problems*, Erdős centennial, 2013. Also available at https://arxiv.org/abs/1306.5167.

Additional support

Office hours will be offered.