Elliptic Partial Differential Equations (L24)

Professor N. Wickramasekera & Dr G. Taujanskas

This course will provide an introduction to the theory of linear second order elliptic partial differential equations. Second order elliptic equations play a fundamental role in many areas of mathematics including Calculus of Variations, Riemannian Geometry and Mathematical Physics. Linear elliptic theory provides the foundation for studying a number of non-linear problems arising in these fields, such as minimal submanifolds, harmonic maps and evolution problems in Geometry and Mathematical Physics.

The course will provide a rigorous treatment, based on a priori estimates, of both classical and weak solutions to linear elliptic equations, focusing on the question of existence and uniqueness of solutions to the Dirichlet problem and the question of regularity of solutions. Specific topics include:

- harmonic functions
- maximum principles for general second order equations
- Schauder estimates (via L. Simon's scaling argument)
- the continuity method for existence of solutions
- solvability of the Dirichlet problem in balls
- Perron's method
- divergence form operators
- De Giorgi–Nash–Moser estimates
- the Harnack theory
- as time permits, a brief discussion of the quasilinear theory centred around the prototypical example of the Minimal Surface Equation.

Prerequisites

Part III *Analysis of PDEs.*

Literature

1. D. Gilbarg and N. Trudinger, *Elliptic partial differential equations of second order.*
2. L. Simon, *Schauder estimates by scaling.* Calc. Var. & PDE, **5**, (1997), 391–407.
3. P. Minter, Lecture notes for the courses *Elliptic PDEs* and *The Minimal Surface Equation and Related Topics*, https://minterscompactness.wordpress.com/lecture-notes/

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.