Complex Manifolds (L24)

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The goal is to introduce the basic theory of complex manifolds. This course relates to both Differential Geometry (a complex manifold is a differentiable manifold endowed with additional structure) and Algebraic Geometry (a non-singular algebraic variety over $\mathbf{C}$ gives an instance of a complex manifold). I expect to cover much of the following:

- *Local analysis and complex structures.* Introduction to several complex variables. Definition and examples of complex manifolds. Holomorphic tangent bundle, (p, q)-forms. Almost complex structures and integrability. Dolbeault cohomology and $\bar{\partial}$-Poincaré lemma.
- *Holomorphic geometry.* Holomorphic line bundles, the Picard group, meromorphic sections. Subvarieties. Divisors and line bundles. Chern connection and the first Chern class. The adjunction formula. Blowing up.
- *Kähler geometry.* Hermitian metrics. Hodge theorem, Serre duality. Kähler manifolds (from several points of view). Kähler identities, Hodge and Lefschetz decompositions. Calculation of basic invariants for hypersurfaces in $\mathbf{CP}^n$. Ricci form and the Calabi–Yau manifolds.

Pre-requisites

Smooth manifolds, tangent bundles, differential forms. Basic knowledge of Riemannian metrics and curvature (including the Ricci curvature). I suppose that all of these will be covered in the course *Differential Geometry* offered in Michaelmas Term which is an ideal pre-requisite.
Basic theory of holomorphic functions in one complex variable will be assumed. Some familiarity with Riemann surfaces will be useful, but not essential.

Literature

[1] D. Huybrechts. *Complex geometry. An introduction.* Springer 2005.

[2] P. Griffiths and J. Harris. *Principles of algebraic geometry*, Wiley 1978.

[3] S. Kobyashi and K. Nomizu. *Foundations of differential geometry*, vol. 2. Wiley 1996.

[4] R.O. Wells. *Differential analysis on complex manifolds.* Springer 1980.

You might like to browse in the first chapter of [1] in advance, the book also has a generous collection of examples. A possible alternative is [2], Ch. 0 and 1 (further chapters may be of interest if you are taking algebraic geometry courses). On the other hand, [4] elaborates on the analysis side of things. The two volumes of [3] are regarded as a canonical reference on differential geometry topics; volume 2 has a chapter on the foundations of complex manifolds.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.