Coxeter Groups (L16)

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Coxeter groups are ubiquitous objects in mathematics, with many connections to algebra, geometry, topology, and representation theory. They are groups generated by reflections satisfying specific relations, and in some cases these groups can be interpreted as reflections across hyperplanes in Euclidean, spherical or hyperbolic space. Dihedral groups (reflection groups of polygons), and symmetric groups (reflection groups of standard simplices) are two examples of families of spherical Coxeter groups.

This course will serve primarily as an introduction to Coxeter groups, and their rich and elegant theory. Along the way, we will encounter tools utilised across many different fields of mathematics. As a whole the course will be primarily algebraic but the tools come from the following fields: algebra, geometric group theory, combinatorics, geometry and topology. The study of Coxeter groups via these five fields can be loosely associated to the five points in the synopsis below.

Course Synopsis:

1. Coxeter systems: definition, examples, Coxeter diagrams.

2. The word problem: Cayley graphs, the exchange condition, Tits' solution to the word problem for Coxeter groups.

3. Combinatorial theory of Coxeter groups: special subgroups, parabolic subgroups, cosets and elements of shortest and longest length.

4. The Tits representation, and classification of finite and affine Coxeter groups: spherical, Euclidean and hyperbolic.

5. The Davis complex and its properties: as a basic construction, as the geometric realisation of a poset, as a CW complex, contractibility, as a classifying space for a family of subgroups.

At the end of the course we may spend some time on one or more of the following related topics: Artin groups, buildings, Weyl groups, cube complexes.

Prerequisites

Linear algebra (e.g. IB Linear Algebra) and a first course on group theory (e.g. IA Groups) is essential. Familiarity with Geometry and Topology (e.g. IB Geometry, II Algebraic Topology) is useful but not essential.

Literature

1. K. Brown *Buildings*. Springer 1989.

2. M. Davis *The Geometry and Topology of Coxeter Groups*. Princeton University Press 2008.

3. J. Humphreys *Reflection Groups and Coxeter groups*. Cambridge University Press 1990.

4. A. Thomas *Geometric and Topological Aspects of Coxeter groups and Buildings*. European Mathematical Society 2018.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.