Numerical solution of differential equations (M24)

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The goal of this lecture course is to present and analyse efficient numerical methods for ordinary and partial differential equations. The exposition is based on few basic ideas from approximation theory, complex analysis, theory of differential equations and linear algebra, leading in a natural progression to a wide range of numerical methods and computational strategies. The emphasis is on algorithms and their mathematical analysis, rather than on applications.

The course consists of three parts:

- methods for **ordinary differential equations** (with an emphasis on initial-value problems and a thorough treatment of stability issues and stiff equations)
- numerical schemes for **partial differential equations** (both boundary and initial-boundary value problems, featuring finite differences and finite elements)
- time allowing, **numerical algebra of sparse systems** (inclusive of fast Poisson solvers, sparse Gaussian elimination and iterative methods).

We start from the very basics, analysing approximation of differential operators in a finite-dimensional framework, and proceed to the design of state-of-the-art numerical algorithms.

Desirable Previous Knowledge

Good preparation for this course assumes relatively little in numerical mathematics *per se,* except for basic understanding of elementary computational techniques in linear algebra and approximation theory. Prior knowledge of numerical methods for differential equations will neither be assumed nor is necessarily an advantage. Experience with programming and application of computational techniques will obviously aid comprehension but is not a prerequisite.

Fluency in linear algebra and decent understanding of mathematical analysis are a must. Thus, linear spaces (inner products, norms, basic theory of function spaces and differential operators), complex analysis (analytic functions, complex integrals, the Cauchy formula), Fourier series, basic facts about dynamical systems and, needless to say, elements from the theory of differential equations.

There are several undergraduate textbooks on numerical analysis. The following present material at a reasonable level of sophistication. Often they present material well in excess of the requirements for the course in computational differential equations, yet their contents (even the bits that have nothing to do with the course) will help you acquire valuable background in numerical techniques:

Introductory Reading

1. S. Conte & C. de Boor, *Elementary Numerical Analysis,* McGraw–Hill, New York, 1980.
2. G.H. Golub & C.F. van Loan, *Matrix Computations,* 3rd edition. Johns Hopkins Press 1996.
3. M.J.D. Powell, *Approximation Theory and Methods,* Cambridge University Press, Cambridge, 1981.

4. G. Strang, *Introduction to Linear Algebra,* Wellesley-Cambridge Press, Cambridge (Mass.), 3rd ed. 2003..

Reading to complement course material

1. U. M. Ascher, *Numerical Methods for Evolutionary Differential Equations,* SIAM, Philadelphia, 2008.
2. E. Hairer, S. P. Nørsett and G. Wanner, *Solving Ordinary Differential Equations I: Nonstiff Problems*, Springer-Verlag, Berlin, 2nd ed. 1993.
3. E. Hairer and G. Wanner, *Solving Ordinary Differential Equations II: Stiff and Differential Algebraic Problems,* Springer-Verlag, Berlin, 2nd ed. 1996.
4. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations*, Cambridge University Press, Cambridge, 2nd ed. 2008.
5. K.W. Morton & D.F. Mayers, *Numerical Solution of Partial Differential Equations: An Introduction,* Cambridge University Press, Cambridge, 2005.
6. G. Strang and G. Fix, *An Analysis of the Finite Element Method*, Wellesley–Cambridge Press, Cambridge (Mass.), 2nd ed. 2008.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.