Topics in Statistical Theory (L16)

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We will develop the mathematical theory underpinning a variety of non-linear statistical inference problems with high- or infinite dimensional parameter spaces. Gaussian non-linear regression, density estimation, and Poisson count models will be covered, and basic applications to inverse problems will be discussed. In such models some kind of regularisation technique is required to deal with the high-dimensionality and non-linearity of the problem, and one can interpret such regularisation methods as posterior inferences arising from certain prior probability distributions. A variety of theorems about the statistical performance of such Bayesian methods will be proved and their optimality will be shown in a variety of settings.

The course will roughly be structured as follows:

I. Non-linear statistical models: Gaussian random design regression models, density estimation, Poisson count data, forward maps and link functions; information distances and inequalities, minimax convergence rates

II. The nonparametric Bayesian approach, the prior as a regularisation technique, posterior contraction rates, convergence rates of posterior mean estimators, application to inverse problems

Pre-requisites

A course on mathematical statistics (such as Part II Principles of Statistics) and measure-theoretic probability theory is essential. Background in functional analysis will be useful but can be acquired during the course. Relevant background material is in [1] and many ideas of the course are contained in [2].

Additional support

Next to the lectures there will be three examples classes and a revision class for this course.

Literature

1. E. Giné & R. Nickl, *Mathematical foundations of infinite-dimensional statistical models*, Cambridge University Press, Cambridge 2016

2. R. Nickl, *Bayesian non-linear statistical inverse problems*, European Mathematical Society Press, Berlin 2023