Theoretical Physics of Soft Condensed Matter (L16)

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Soft Condensed Matter refers to liquid crystals, emulsions, colloidal suspensions and other microstructured fluids or semi-solid materials. Alongside many high-tech examples, domestic and biological instances include mayonnaise, toothpaste, engine oil, shaving cream, and the lubricant that stops our joints scraping together. Their behaviour is classical ($\hbar = 0$) but rarely is it deterministic: thermal noise is generally important.

The basic modelling approach therefore involves continuous classical field theories, generally with noise so that the equations of motion are stochastic PDEs. The form of these equations is helpfully constrained by a requirement that the Boltzmann distribution is regained in the steady state. Both the dynamical and steady-state behaviours have a natural expression in terms of path integrals, defined as weighted sums of trajectories (for dynamics) or configurations (for steady state). These concepts will be introduced in a relatively informal way, focusing on how they can be used for actual calculations.

In many cases mean-field treatments are sufficient, simplifying matters considerably. But we will also meet examples such as the phase transition from an isotropic fluid to a 'smectic liquid crystal' (a layered state which is periodic, with solid-like order, in one direction but can flow freely in the other two). Here mean-field theory gets the wrong answer for the order of the transition, but the right one is found in a self-consistent treatment that lies one step beyond mean-field (and one step short of the renormalization group, which we leave to the Statistical Field Theory course).

Important dynamical models of soft matter include diffusive ϕ^4 field theory ('Model B'), and the noisy Navier-Stokes equation which describes fluid mechanics at colloidal scales, where the noise term is responsible for Brownian motion of suspended particles in a fluid. Coupling these together creates 'Model H', a theory that describes the physics of fluid-fluid mixtures (that is, emulsions). We will explore Model B, and then Model H, in some depth. We will also explore the continuum theory of nematic and polar liquid crystals, which spontaneously break rotational but not translational symmetry, focusing on topological defects and their associated mathematical structure such as homotopy classes.

Finally, the course will briefly analyse soft-matter systems whose microscopic dynamics involves the continuous dissipation of energy, such as self-propelled colloidal swimmers. We will discuss how their absence of time-reversal symmetry leads to qualitative changes in dynamical behaviour.

Prerequisites

Knowledge of Statistical Mechanics at an undergraduate level is essential. It is not necessary to attend any other particular part III courses but there is potential synergy with Statistical Field Theory, Slow Viscous Flow, Stochastic Processes in Biology, and to a lesser extent some other courses too. For field theorists, this is a good opportunity to see what statistical field theory looks like when addressing time evolution rather than the purely static properties set by the Boltzmann distribution. For people interested in fluid mechanics, it is a good opportunity to see what this looks like beyond Navier-Stokes fluids, in systems with thermal noise and/or structural order parameters. In previous years the audience has included a mix of students whose main specialism is either field theory or fluid dynamics. People with these differing backgrounds may find different parts of the course easier or harder, but the intention is to create a roughly level playing field so that all can enjoy learning about this interdisciplinary field.

Literature

1. D. Tong, *Lectures on Statistical Physics*

   http://www.damtp.cam.ac.uk/user/tong/statphys.html

   Before embarking on this course you need to understand the equation $F = -k_B T \ln Z$ and its implications. This includes knowing what the Boltzmann distribution is, what it describes, and when it is true. You should also have met the concept of chemical potential and the grand canonical ensemble. Acquaintance with the Landau theory of phase transitions is helpful. David Tong's lecture notes are an excellent resource for revising and reviewing the key material.

2. M. E. Cates and E. Tjhung, *Theories of binary fluid mixtures: from phase-separation kinetics to active emulsions.* J. Fluid Mech. (2018), **836**, pp 1-66.

   https://www.cambridge.org/core/journals/journal-of-fluid-mechanics/article/theories-of-binary-fluid-mixtures-from-phaseseparation-kinetics-to-active-emulsions/5BD133CB20D89F47E724D77C296FEF80/share/106fd30f307db12134745de39fd568fbbaa3f9d2

   This JFM perspectives article has significant overlap (perhaps 40%) with the course but takes fluid mechanics as its starting point whereas we will start from statistical physics and bring in fluid mechanics as needed. It gives a flavour of the types of problem we will address and some of the methodologies involved. However we will not have time to cover much of the material it contains on active systems.

3. P. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics.* Cambridge University Press, 1995.

   An authoritative and broad ranging but advanced book, that is worth dipping into to see how hydrodynamics, broken symmetries, and topological defects all feature in the description of condensed matter systems at $\hbar = 0$. This is more for inspiration than information though; this course may help you in understanding the book, but probably not vice versa.

4. Unofficial lecture notes were taken several years ago:

   https://dec41.user.srcf.net/h/III_L/theoretical_physics_of_soft_condensed_matter

   These notes are far from perfect but are mentioned here so that all students are aware of their existence.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a 90-minute revision class in the Easter Term. The lecturer will be available at other times by arrangement.

Note on past papers

Starting in the 2016/7 academic year, this course was given in 16-lecture format for 3 years, then as a 24-lecture course for 2 years, before reverting to 16 lectures from the 2021/2 academic year onwards. This means the summer 2021 tripos paper examined some material that lies beyond the current syllabus. (There was no honours paper in 2020.)