Gravitational Waves and Numerical Relativity (E16)

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The direct detection of a gravitational-wave signal from the black-hole binary merger GW150914, detected by LIGO in September 2015, has promoted gravitational-wave physics to a rich field combining theory with observation. With about 100 further events observed over the ensuing years, a vast range of physical and astrophysical phenomena are now accessible to a qualitatively new channel of observations. New gravitational-wave observatories on Earth and in Space will greatly enhance the sensitivity and frequency range in future decades. Gravitational-wave observations heavily rely on accurate theoretical predictions that require source modelling in the framework of Einstein's theory of relativity. Due to the complexity of Einstein's field equations and the highly dynamical character of the physical systems under consideration, the calculation of such templates is a formidable challenge that requires the combination of analytic and numerical techniques. In this lecture course, we will discuss the theoretical foundations of gravitational waves (at linear and non-linear level), formulations of the Einstein field equations amenable for a numerical treatment, initial data, gauge conditions and diagnostic tools.

More specifically, the course will cover the following topics.

- Characteristic surfaces and the classification of partial differential equations.
- The structure of the Einstein field equations.
- Gravitational waves in the characteristic (aka *Bondi-Sachs*) formalism of general relativity.
- The space-time split of the Einstein equations and well-posed formulations of the field equations.
- The construction of constraint satisfying initial data representing realistic snapshots of systems of multiple black holes.
- Singularity avoiding coordinate choices employed in contemporary numerical relativity simulations.
- Diagnostic tools to extract the gravitational-wave signal from numerically generated solutions.

Prerequisites

This course assumes knowledge of Part III General Relativity. Some knowledge of Part III Black Holes will be helpful but not essential for following this course.

Literature

1. T. W. Baumgarte and S. L. Shapiro *Numerical Relativity.* Cambridge University Press, 2010.
2. T. W. Baumgarte and S. L. Shapiro *Numerical Relativity: Starting from Scratch.* Cambridge University Press, 2021.
3. M. Alcubierre, *Introduction to 3+1 Numerical Relativity*, Oxford University Press, 2008.

4. E. Gourgoulhon, *3+1 Formalism and Bases of Numerical Relativity*, Springer, New York, 2012. See also https://arxiv.org/abs/gr-qc/0703035.

5. M. Maggiore, Gravitational Waves. Vols. 1 (Theory and Experiments) and 2 (Astrophysics and Cosmology), Oxford University Press, 2018.

6. R. d'Inverno, *Introducing Einstein's Relativity*, Oxford Clarendon Press, 1992.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.