Solitons, instantons and geometry (L16)

Professor D. Stuart

In classical field theory, solutions which are stable, spatially localized and particle-like are called *solitons*; they exist in many nonlinear field theories. In many quantum field theories, including gauge theories, it is necessary to consider solutions of the field equations that are topologically distinct from the vacuum: solitons of this type are known as topological solitons. We will discuss existence of solitons in various examples, and describe their properties and dynamics.

In a slightly different direction, classical solutions which are also localised in *Euclidean* space-time are called *instantons*, and are studied in connection with tunneling phenomena and the vacuum in field theory.

In certain cases, the (usually) second order field equations can be reduced to first order Bogomolny equations, which makes it easier to analyse the soliton/instanton solutions. The resulting equations are often integrable in some sense, and explicit solutions can sometimes be found. Examples include scalar kinks, magnetic monopoles in Yang-Mills-Higgs theory and anti-self-dual instantons in pure Yang-Mills theory.

An introduction to the mathematical framework behind gauge theory (vector bundles, connections and curvature) will be given, together with some basic ideas from topology and calculus of variations as needed.

Prerequisites

Part II Quantum Mechanics, Electrodynamics and General Relativity (or some introductory Differential Geometry course such as the one in Part II) are essential. Part III General Relativity and Quantum Field Theory will be helpful.

Literature

1. T.D. Lee, *Particle physics and introduction to field theory,* Harwood, 1981. Chapter 7. (A good introduction to the phenomena).

2. R. Rajaraman, *Solitons and instantons, An introduction to solitons and instantons in quantum field theory.* North-Holland Publishing Co. 1982. (As well as general introduction and examples similar to Lee this contains material on semiclassical quantization of solitons.)

3. A. Jaffe and C. Taubes, *Vortices and monopoles,* Birkhauser, 1982. Chapters 1,2 and 3. (A mathematical treatment for those interested in analysis).

4. N.S. Manton and P.M. Sutcliffe, *Topological Solitons,* CUP, 2004, Chapters 3,4,5,6,8 and 10. (Detailed discussion of the physics of solitons, beyond this course.)

5. M. Dunajski, *Solitons, Instantons, and Twistors,* Oxford Graduate Texts in Mathematics, Oxford University Press, 2009. (Includes introduction to integrable aspects from the twistor point of view.)

6. T. Eguchi, P. Gilkey, and A.J. Hanson. *Physics Reports,* 66 (1980) 213-393. (Mathematical background.)

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.