Solitons, Instantons, and Geometry (L16)

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Solitons are solutions of classical field equations with particle-like properties. They are localised in space, have finite energy and are stable against decay into radiation. The stability usually has a topological explanation. Instantons are non–singular solutions of classical field equations in Euclidean space whose action is finite. Instantons describe quantum-mechanical tunnelling between the classical minima of the potential in Minkowski space. Solitons and instantons are globally characterised by some topological charge, related to the behaviour of solutions at spatial infinity. Its understanding requires ideas from topology, and geometry.

The course will cover a selection of topics including:

- Solitons (kinks in 1D, vortices in 2D, monopoles in 3D)
- Yang Mills instantons, and their integrability.
- Fibre bundles and instantons: connection, curvature and characteristic classes.

Prerequisites

Basic General Relativity (Part II level) or some introductory Differential Geometry course (e.g. Part II differential geometry) is essential. Part III General Relativity is desirable.

Literature

1. Dunajski, M. Solitons, Instantons, and Twistors. (2nd Edition) Oxford Graduate Texts in Mathematics **31**, Oxford University Press, 2024.
2. Eguchi, T., Gilkey, P. and Hanson. A. J. Physics Reports **66** (1980) 213-393
3. Manton, N. and Sutcliffe, P. Topological Solitons. Cambridge University Press, 2004.

Additional support

Three examples sheets will be provided and three examples classes will be given. There will be a one-hour revision class in the Easter Term.