Canonical Gravity (L16)

Non-Examinable (Graduate Level)

Professor M.J. Perry

General relativity is often presented in a way that unifies space and time into a complete whole, spacetime. This approach is encapsulated in the Einstein equation

$$R_{ab} - \frac{1}{2} R\, g_{ab} = 8\pi G T_{ab} \tag{1}$$

where there is no separation into space and time. Nevertheless, physics is often formulated in terms of the initial value problem where the initial configuration of spatial geometry, fields and particles together with their time derivatives are specified. One can then compute the time evolution of the system. We apply these ideas to general relativity. Firstly, we need to understand the general picture of constrained Hamiltonian systems and why the idea of constraints is related to gauge degrees of freedom. We will prqctice these ideas for particle mechanics, scalar field theory and then electromagnetism. Finally we apply these methods to general relativity to provide a canonical formulation along the lines first found by Dirac and by Arnowitt, Deser and Misner. More generally, we will then examine the evolution of a spacetime in terms of its trajectories in superspace, the space of all spatial geometries. We then will apply these ideas to a simple cosmological model and subsequently look how these ideas impact the problem of singularities is spacetime and the chaotic behaviour associated with them.

Quantum mechanics is usually presented starting from a Hamiltonian and then developed as a theory in which classical phase space is deformed using the Heisenberg algebra in which classical quantities are replaced by operators. We apply this technique to general relativity and so develop the Wheeler-DeWitt equation. Then we can use this to see how a classical spacetime can emerge from the quantum picture by use of the WKB approximation. Finally, if time permits, we will look at the relationship between the Wheeler-DeWitt equation and the path integral, what happens when the Wheeler-DeWitt equation is used to examine spacetime singularities, and lastly the role of observers to extract measurements in quantum gravity.

Prerequisites

Basic general relativity. Quantum Mechanics.

Additional support

There will be no examples sheets. An office hour will be arranged but perhaps some kind of discussion session can be organised.