Topological Field Theory in Quantum Matter

Non-Examinable (Graduate Level)

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The study of topological phases of quantum matter has benefited from its connections to topological field theory (TFT). A partial dictionary between the two structures allows mathematicians to translate concepts from one picture to the other, where new insights can be gleaned. The correspondence is more of an intuition than a rigorous statement (and recent studies have identified exceptions to it), but there are settings where it can be made precise. This essay aims to understand the connection between topological phases and TFT through concrete constructions.

The course begins with an introduction to TFT and Atiyah's axioms, which are used to classify TFTs in low dimensions. Famously, 2d TFTs are described by commutative Frobenius algebras.

We will then discuss state sum constructions. The idea of a state sum is to discretize partition functions as products of local amplitudes on the simplices that triangulate spacetime. Then, by imposing local relations on these amplitudes, partition functions can be made to be independent of the triangulation; that is, made to be topological. We will give special attention to Dijkgraaf-Witten theory, which is a state sum model for topological gauge theory. Another point of focus will be the state sum construction of the 2d TFTs examined previously.

Finally we will turn to 2+1d topological lattice models such as quantum double and string-net models. Our discussion will center on the relationship between topological state sums and the projected entangled pair state (PEPS) tensor networks that describe these models.

Prerequisites

It will be assumed that you have taken an advanced quantum mechanics course similar to Part II Principles of Quantum Mechanics. This course is designed to complement the discussion of topological quantum many-body physics made in the Topological Quantum Matter and Quantum Entanglement in Many-body Physics courses in Lent term; basic familiarity with the key concepts from these courses will provide useful context but is not required.

Literature

1. M. Atiyah, *An Introduction to Topological Quantum Field Theory.* Michael Atiyah Collected Works, p. 265 (2005).

2. S. Fukuma, A. Hosono, H. Kawai, *Lattice Topological Field Theory in two dimensions*, Comm. Math. Phys. 161, no. 1, 157–175 (1994).

3. R. Dijkgraaf, E. Witten, *Topological gauge theories and group cohomology*, Communications in Mathematical Physics 129 393–429 (1990).

4. J. Barrett, B. Westbury, *Invariants of piecewise-linear 3-manifolds*, Trans. Amer. Math. Soc. 348, no. 10, 3997–4022 (1996).

5. A. Kirillov Jr, *String-net model of Turaev-Viro invariants*, arXiv: 1106.6033 (2011).

6. A. Kapustin, A. Turzillo, M. You, *Topological Field Theory and Matrix Product States*, Phys. Rev. B 96, 075125 (2017)

7. L. Lootens, J. Fuchs, J. Haegeman, C. Schweigert, F. Verstraete, *Matrix product operator symmetries and intertwiners in string-nets with domain walls*, SciPost Phys. 10, 053 (2021)

Additional support

Accompanying office hours will be offered.