Random discrete structures (M16)

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A phase transition means that a system undergoes a radical change when a continuous parameter passes through a critical value. We encounter such a transition every day when we boil water. The simplest mathematical model for phase transition is percolation. Percolation has a reputation as a source of beautiful mathematical problems that are simple to state but seem to require new techniques for a solution, and a number of such problems remain very much alive. Amongst connections of topical importance are the relationships to so-called Schramm-Loewner evolutions (SLE), and to other models from statistical physics. The basic theory of percolation will be described in this course with some emphasis on areas for future development.

Our other major topic includes random walks on graphs and their intimate connection to electrical networks; the resulting discrete potential theory has strong connections with classical potential theory. We will develop tools to determine transience and recurrence of random walks on infinite graphs. Other topics include the study of spanning trees of connected graphs. We will present two remarkable algorithms to generate a uniform spanning tree (UST) in a finite graph G via random walks, one due to Aldous-Broder and another due to Wilson. These algorithms can be used to prove an important property of uniform spanning trees discovered by Kirchhoff in the 19th century: the probability that an edge is contained in the UST of G, equals the effective resistance between the endpoints of that edge.

Pre-requisites

There are no essential pre-requisites beyond probability and analysis at undergraduate levels, but a familiarity with the measure-theoretic basis of probability will be helpful.

Literature

1. Bollobás, B. and Riordan, O., *Percolation*, Cambridge University Press, 2006

2. Grimmett, G. R., *Probability on Graphs*, Cambridge University Press, 2010 available at http://www.statslab.cam.ac.uk/~grg/books/pgs.html

3. Grimmett, G. R., *Percolation*, Springer-Verlag, Berlin, second edition, 1999

4. Lyons, R. and Peres, Y., *Probability on Trees and Networks*, available at https://rdlyons.pages.iu.edu/prbtree/book_pb.pdf