Stochastic Calculus (L24)

Professor J. Miller

This course will be an introduction to Itô calculus and will aim to cover the following topics.

- *Brownian motion.* Existence and sample path properties.
- *Stochastic calculus for continuous processes.* Martingales, local martingales, semi-martingales, quadratic variation and cross-variation, Itô's isometry, definition of the stochastic integral, Kunita-Watanabe theorem, and Itô's formula.
- *Applications to Brownian motion and martingales.* Lévy characterization of Brownian motion, Dubins-Schwartz theorem, martingale representation, Girsanov theorem, conformal invariance of planar Brownian motion, and Dirichlet problems.
- *Stochastic differential equations.* Strong and weak solutions, notions of existence and uniqueness, Yamada-Watanabe theorem, strong Markov property, and relation to second order partial differential equations.
- *Stroock–Varadhan theory.* Diffusions, martingale problems, equivalence with SDEs, approximations of diffusions by Markov chains.

Prerequisites

We will assume knowledge of measure theoretic probability as taught in Part III Advanced Probability. In particular we assume familiarity with discrete-time martingales and Brownian motion.

Literature

1. R. Durrett *Probability: theory and examples.* Cambridge, 2010.
2. I. Karatzas and S. Shreve *Brownian Motion and Stochastic Calculus.* Springer, 1998.
3. P. Morters and Y. Peres *Brownian Motion.* Cambridge, 2010.
4. D. Revuz and M. Yor, *Continuous martingales and Brownian motion.* Springer, 1999.
5. L.C. Rogers and D. Williams *Diffusions, Markov Processes, and Martingales.* Cambridge, 2000.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.