Stochastic Calculus (L24)

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This course will be an introduction to Itô calculus and will aim to cover a selection of the following topics.

- *Brownian motion.* Existence and sample path properties.
- *Stochastic calculus for continuous processes.* Martingales, local martingales, semi-martingales, quadratic variation and cross-variation, Itô's isometry, definition of the stochastic integral, Kunita-Watanabe theorem, and Itô's formula.
- *Applications to Brownian motion and martingales.* Lévy characterization of Brownian motion, Dubins-Schwartz theorem, martingale representation, Girsanov theorem, conformal invariance of planar Brownian motion, and Dirichlet problems.
- *Stochastic differential equations.* Strong and weak solutions, notions of existence and uniqueness, Yamada-Watanabe theorem, strong Markov property, and relation to second order partial differential equations.
- *Stroock–Varadhan theory.* Diffusions, martingale problems, equivalence with SDEs, approximations of diffusions by Markov chains.

Pre-requisites

We will assume knowledge of measure theoretic probability as taught in Part III Advanced Probability. In particular we assume familiarity with discrete-time martingales and Brownian motion.

Literature

1. R. Durrett *Probability: theory and examples.* Cambridge. 2010
2. I. Karatzas and S. Shreve *Brownian Motion and Stochastic Calculus.* Springer. 1998
3. P. Morters and Y. Peres *Brownian Motion.* Cambridge. 2010
4. D. Revuz and M. Yor, *Continuous martingales and Brownian motion.* Springer. 1999
5. L.C. Rogers and D. Williams *Diffusions, Markov Processes, and Martingales.* Cambridge. 2000