Schramm-Loewner Evolutions (L16)

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Schramm-Loewner evolution (SLE), a family of random fractal curves on planar domains, was introduced in the early 2000s to describe scaling limits of interfaces in critical two-dimensional lattice models (such as the Ising model, percolation, Gaussian free field...). Soon it became apparent that it plays a major role in two-dimensional probability (more precisely, conformally invariant processes). One famous example is that SLE arguments have been successfully applied to prove several conjectures about two-dimensional Brownian motion and simple random walks (intersection exponents, Mandelbrot conjecture). Of course, SLE is highly interesting in its own right, combining ideas from complex analysis and probability, and giving rise to beautiful mathematics.

The main goal of this course is to define and study SLE. The following topics will be covered:

- conformal maps: relations to planar Brownian motion, distortion estimates, and the Loewner differential equation,
- the definition of SLE, basic properties and their geometry,
- relation to the Gaussian free field.

Prerequisites

To follow this course, the student should be familiar with

- measure-theoretic probability (as taught in the course Advanced Probability),
- basic complex analysis, in particular conformal maps and the Riemann mapping theorem,
- stochastic calculus (can be attended in parallel to this course).

Literature

The course will follow the lecture notes by Jason Miller: http://www.statslab.cam.ac.uk/~jpm205/teaching/lent2019/sle_notes.pdf.

For extra material on SLE the student can consult:

1. Wendelin Werner, *Random planar curves and Schramm-Loewner evolutions*, 2004. Also available at https://arxiv.org/abs/math/0303354.
2. Gregory F. Lawler, *Conformally Invariant Processes in the Plane*, volume 114 of Mathematical Surveys and Monographs. American Mathematical Society, Providence, RI, 2005.

For extra material on conformal maps, see the following: (Note that the references below are beyond the prerequisites of the course. We will cover what is needed but the interested student will enjoy the books.)

1. Dimitry Beliaev, *Conformal maps and geometry*, Advanced Textbook in Mathematics. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2020.
2. John B. Garnett and Donald Marshall, *Harmonic Measure*, New Mathematical Monographs: 2. CUP, 2005.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.