Percolation and related topics (M16)

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What is the probability that the centre of a large porous stone immersed in water is wetted? To answer this question, Broadbent and Hammersley (1957) introduced the beautiful mathematical model of percolation. Percolation is renowned for generating elegant problems that are easy to formulate but rather hard to settle, and many of these questions remain open today. The occurrence of a 'critical phenomenon' is central to percolation. This course will describe the basic theory of percolation, with emphasis on phase transitions, subcritical and supercritical phases, size of open clusters, among other things.

The second half of the course will cover a special class of percolation model called last passage percolation, which is currently one of the most active research areas in probability. It is the model of a directed path in a random environment that takes the longest time to reach a given point. We will talk about the famous Ulam–Hammersley problem of finding the maximal length of an increasing subsequence in a uniformly random permutation of the first n natural numbers, and its connection to Poissonian last passage percolation. Subject to time constraints, we will also consider other related models, including exponential and Brownian last-passage percolation, as well as their scaling limits—the KPZ fixed point and the directed landscape.

Prerequisites

There are no essential pre-requisites beyond probability and analysis at undergraduate levels, but a familiarity with measure-theoretic probability will be helpful. It is also useful to follow the Advanced Probability course concurrently in the Michaelmas term.

Literature

1. Grimmett, G. R., *Probability on Graphs.* Cambridge University Press, 2010 available at http://www.statslab.cam.ac.uk/~grg/books/pgs.html.

2. Grimmett, G. R., *Percolation.* Springer-Verlag, Berlin, second edition, 1999.

3. Romik, D., *The Surprising Mathematics of Longest Increasing Subsequences.* Cambridge University Press, 2015.

Additional support

Three examples sheets will be provided and three associated examples classes will be given.