Lattice Models (M16)

Professor W. Werner

Lattice models are probabilistic models defined on $\mathbf{Z}^d$ (or other lattices). Examples include the Ising model, percolation, the Potts models, dimer models, the self-avoiding walk, uniform spanning trees and forests, the Gaussian Free Field and others. Most of these models are motivated from statistical physics. They are all simple to define, yet they often have very rich behaviour, and a number of fundamental and easy-to-state questions remain open.

This course will present a small selection of ideas and results. We will discuss aspects of the phase transition for percolation, Wilson's algorithm to construct uniform spanning trees, and properties of the discrete Gaussian Free Field. We will also describe some features specific to models in the plane, such as Smirnov's proof of conformal invariance for percolation on the triangular lattice.

Prerequisites

This course assumes familiarity with probability, measure theory, and analysis at Part II level. Parallel participation in Advanced Probability is encouraged.

Literature

The course will use material from various sources. Some helpful references are:

1. H. Duminil-Copin, *Introduction to percolation theory.*
   Availiable here: https://www.unige.ch/~duminil/publi/2017percolation.pdf
2. G. Grimmett, *Probability on Graphs*, Cambridge University Press
   Available here: http://www.statslab.cam.ac.uk/~grg/books/pgs.html
3. W. Werner and E. Powell, Lecture notes on the Gaussian Free Field, Cours Spécialisés S.M.F.
   Available here: https://arxiv.org/abs/2004.04720

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term..