Advanced Probability (M24)

Professor P. Sousi

The aim of the course is to introduce students to advanced topics in modern probability theory. The emphasis is on tools required in the rigorous analysis of stochastic processes, such as Brownian motion, and in applications where probability theory plays an important role.

- **Review of measure and integration:** sigma-algebras, measures and filtrations; integrals and expectation; convergence theorems; product measures, independence and Fubini's theorem.
- **Conditional expectation:** Discrete case, Gaussian case, conditional density functions; existence and uniqueness; basic properties.
- **Martingales:** Martingales and submartingales in discrete time; optional stopping; Doob's inequalities, upcrossings, martingale convergence theorems; applications of martingale techniques.
- **Stochastic processes in continuous time:** Kolmogorov's criterion, regularization of paths; martingales in continuous time.
- **Weak convergence:** Definitions and characterizations; convergence in distribution, tightness, Prokhorov's theorem; characteristic functions, Lévy's continuity theorem.
- **Sums of independent random variables:** Strong laws of large numbers; central limit theorem; Cramér's theory of large deviations.
- **Brownian motion:** Wiener's existence theorem, scaling and symmetry properties; martingales associated with Brownian motion, the strong Markov property, hitting times; properties of sample paths, recurrence and transience; Brownian motion and the Dirichlet problem; Donsker's invariance principle.
- **Poisson random measures:** Construction and properties; integrals.
- **Lévy processes:** Lévy-Khinchin theorem.

Prerequisites

A basic familiarity with measure theory and the measure-theoretic formulation of probability theory is very helpful. These foundational topics will be reviewed at the beginning of the course, but students unfamiliar with them are expected to consult the literature (for instance, Williams' book) to strengthen their understanding.

Literature

1. Lecture notes available online
2. D. Applebaum, *Lévy processes* 2nd ed. Cambridge University Press, 2009.
3. R. Durrett, *Probability: Theory and Examples* 4th ed. CUP, 2010.
4. O. Kallenberg, *Foundations of Modern Probability.* Springer-Verlag, 1997.
5. D. Williams, *Probability with martingales.* CUP, 1991.

Additional support

Four example sheets will be provided along with associated supervisions or examples classes. There will be a revision class in Easter term.