Symmetries, Particles, and Fields (M24)

M B Wingate

Lie groups and Lie algebras are important in the construction of quantum field theories which describe interactions between particles. Particle states and quantum fields are described in terms of irreducible representations of the Poincaré group, a Lie group. Gauge theories, which describe interactions in the Standard Model of particle physics, rely on Lie groups.

This course will introduce the mathematics necessary for employing the Lie groups and algebras in high energy physics and beyond. The focus will be on the mathematical concepts and key applications. Mathematical proofs will be given where straightforward or useful for a deeper understanding; however, proof of some facts will remain beyond the scope of this course.

After preliminary remarks, we introduce Lie matrix groups, which depend on continuous parameters. Differentially, these act as a Lie algebra. The exponential map connects the Lie algebra to the Lie group. We then introduce group and algebra representations as linear maps on vector spaces.

The group of rotations in 3-dimensional space, $SO(3)$, is examined, along with $SU(2)$ and the connection to angular momentum states in quantum theory. Representations of these are covered and used later in the course. The relativistic symmetries (Lorentz group and Poincaré group in four dimensions) are studied from the point of view of their group elements and Lie algebras. We shall cover irreducible induced representations of the Poincaré group, which are used to describe one-particle states and fields.

Analysis of simple Lie algebras and their finite representations comes from mapping them to a geometrical picture involving roots and weights via the Cartan matrix. An overview of the results of the Cartan classification of simple Lie algebras is included.

An application in terms of representations of a global $SU(3)_F$ flavour symmetry explains some features of the spectrum of hadronic particles. Further properties of the spectrum lead one to introduce an additional local $SU(3)_c$ colour symmetry leading a particular gauge theory called quantum chromodynamics. We shall close by covering gauge theory for general Lie groups and different representations of matter.

Prerequisites

Linear algebra, including direct sums and tensor products of vector spaces. Special relativity and quantum theory, including angular momentum theory and Pauli spin matrices. Brief reminders of some definitions are given; however familiarity with undergraduate quantum mechanics will be important for fully appreciating some of the physical applications of the mathematics in this course.

Literature

1. For a more basic introduction to the use of groups in physics, *Group Theory in a Nutshell for Physicists*, A. Zee (Princeton University Press, 2016).

2. *Lie Algebras in Particle Physics* by H. Georgi (Westview Press, 1999) gives an introduction aimed at physicists.

3. *Groups, Representations, and Physics*, H. F. Jones (Taylor and Francis, 1998). The first half covers discrete groups; later chapters cover some material in this course.

4. *Lie Groups, Lie Algebras, and Representations: An Elementary Introduction*, B. C. Hall (Springer, 2015). A more mathematical text, useful reference for detail beyond this course.

5. *Symmetries and Group Theory in Particle Physics*, G. Costa and G. Fogli (Springer, 2012) has nice geometrical aspects.

Additional support

Four Examples Sheets will be provided and four associated Examples Classes will be given (the final one in Lent term). There will be a Revision Lecture in the Easter Term.