Symmetries, Particles and Fields (M24)

Ben Allanach

Lie algebras and Lie groups are important in the construction of quantum field theories which describe interactions between known particles.

After some preliminaries, we introduce matrix Lie groups which rely on continuous parameters. Differentially, these act as a Lie algebra. The exponential map connects the Lie algebra to the Lie group.

We then introduce representations in terms of square matrices, describing how to construct various new representations in terms of others.

The group of rotations in three-dimensional space $SO(3)$ is examined, along with $SU(2)$ and the connection to angular momentum states in quantum theory. Representations of each are covered.

The relativistic symmetries (Lorentz group and Poincaré group in four dimensions) are studied from the point of view of their group elements and Lie algebras. Representations of the Poincaré group are used to describe one-particle states and fields.

Analysis of compact simple Lie algebras and their finite representations comes from a geometrical picture involving via the Cartan matrix. An overview of the results of the Cartan classification of simple Lie algebras is included. An application in terms of representations of a global $SU(3)_F$ flavour symmetry explains some features of the spectrum of hadronic particles. Further properties of the spectrum lead one to introduce an additional local $SU(3)_c$ colour symmetry leading to a particular gauge theory called quantum chromodynamics.

We shall close by covering gauge theory: first $U(1)$, then general Lie groups and different representations of matter.

Prerequisites

Linear algebra including direct sums and tensor products of vector spaces. Special relativity and quantum theory, including orbital angular momentum theory and Pauli spin matrices.

Literature

1. *Symmetries, Particles, and Fields*, B.C. Allanach, Kindle Direct Publishing, 2025 will form the printed and extended course notes.
2. *Symmetries and Group Theory in Particle Physics*, G. Costa and G. Fogli, Springer, 2012 has nice geometrical aspects.
3. For a more basic introduction to the use of groups in physics, *Group Theory in a Nutshell for Physicists*, Zee, Princeton University Press, 2016.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.