Symmetries, Particles, and Fields (M24)

Professor M B Wingate

Lie groups and Lie algebras are important in the construction of quantum field theories which describe interactions between known particles. Particle states and quantum fields are described in terms of irreducible representations of the Poincaré group, a Lie group. Gauge theories, which describe interactions in the Standard Model of particle physics, rely on Lie groups.

After preliminary remarks, we introduce Lie matrix groups, which depend on continuous parameters. Differentially, these act as a Lie algebra. The exponential map connects the Lie algebra to the Lie group. We then introduce representations in terms of square matrices.

The group of rotations in 3-dimensional space, $SO(3)$, is examined, along with $SU(2)$ and the connection to angular momentum states in quantum theory. Representations of each are covered. The relativistic symmetries (Lorentz group and Poincaré group in four dimensions) are studied from the point of view of their group elements and Lie algebras.

Analysis of simple Lie algebras and their finite representations comes from mapping them to a geometrical picture involving roots and weights via the Cartan matrix. An overview of the results of the Cartan classification of simple Lie algebras is included.

An application in terms of representations of a global $SU(3)_F$ flavour symmetry explains some features of the spectrum of hadronic particles. Further properties of the spectrum lead one to introduce an additional local $SU(3)_c$ colour symmetry leading a particular gauge theory called quantum chromodynamics. We shall close by covering gauge theory for general Lie groups and different representations of matter.

Prerequisites

Linear algebra, including direct sums and tensor products of vector spaces. Special relativity and quantum theory, including orbital angular momentum theory and Pauli spin matrices.

Literature

1. G. Costa and G. Fogli, *Symmetries and Group Theory in Particle Physics*, Springer, 2012 has nice geometrical aspects.

2. For a more basic introduction to the use of groups in physics, Zee, *Group Theory in a Nutshell for Physicists*, Princeton University Press, 2016.

3. H. F. Jones, Taylor and Francis, *Groups, Representations, and Physics*, 1998. The first half covers discrete groups; later chapters cover some material in this course.

4. B. C. Hall, *Lie Groups, Lie Algebras, and Representations: An Elementary Introduction*, Springer, 2015. A more mathematical text, useful reference for detail beyond this course.

Additional support

Four Examples Sheets will be provided and four associated Examples Classes will be given (the final one in Lent term). There will be a one-hour Revision Class in the Easter Term.