Quadratic Forms (L16)

Non-Examinable (Graduate Level)

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A quadratic form over a ring R is a homogeneous polynomial of degree 2 in some number of variables $x_1, \ldots, x_n$, with coefficients in R. The study of quadratic forms over arithmetic rings R (such as number fields or their completions, or their rings of integers) is a beautiful and classical topic in its own right, but also serves as an entry point into many important parts of number theory, including the theory of algebraic and arithmetic groups and the theory of automorphic forms. In this course we will develop the theory of quadratic forms from scratch, focusing on the classification of quadratic forms over the rings $\mathbf{Q}$ and $\mathbf{Z}$.

Topics to be covered include:

- Classification of quadratic forms over $\mathbf{Q}_p$, $\mathbf{R}$, and $\mathbf{Q}$.

- Theorem of Hasse–Minkowski.

- Theory over $\mathbf{Z}$ and $\mathbf{Z}_p$: lattices, genus theory.

- Reduction theory and finiteness of the class number.

Prerequisites

I will assume familiarity with the theory of symmetric bilinear forms (at the level of IB Linear Algebra) and p-adic numbers (at the level of Part III Local Fields).

Literature

1. Serre, J.-P., *A course in arithmetic.* Graduate Texts in Mathematics, No. 7. Springer-Verlag, 1973.

2. Cassels, J. W. S., *Rational quadratic forms.* London Mathematical Society Monographs, 13, Academic Press, Inc., 1978.

3. O'Meara, O. T., *Introduction to quadratic forms.* Classics in Mathematics. Springer-Verlag, 2000.