Modular Forms (M24)

Professor J. Thorne

Modular forms are holomorphic functions on the complex upper half plane which are invariant under an action of the group $SL_2(\mathbf{Z})$ (or a finite index subgroup). They feature in many different parts of mathematics, but are most famous in number theory for their role in the proof of Fermat's Last Theorem, itself based on a proof of the Shimura–Taniyama conjecture concerning the modularity of elliptic curves over $\mathbf{Q}$.

In this course we will introduce the theory of modular forms from a number-theoretic point of view, including the theory of Eisenstein series, Hecke operators, and the connection with L-functions.

Prerequisites

Essential: IB Complex Analysis , IB Groups, Rings & Modules (or equivalents)

Useful: Part II Riemann Surfaces (or equivalent)

Literature

1. J.-P. Serre, *A Course in Arithmetic*, Graduate Texts in Maths. 7, Springer, New York, 1973

2. F. Diamond, J. Shurman, *A First Course in Modular Forms*, Graduate Texts in Maths. 228, Springer, New York, 2005

3. J. Milne, *Modular Functions and Modular Forms*, Lecture notes available at

   https://www.jmilne.org/math/CourseNotes/MF.pdf

4. M. Ram Murty, Applications of symmetric power L-functions. In *Lectures on automorphic L-functions*, pp. 203–283, Fields Inst. Monogr., 20, Amer. Math. Soc., Providence, RI, 2004

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.