

Local Fields (M24)

Rong Zhou

The p -adic numbers \mathbb{Q}_p were introduced by Hensel at the end of the 19th century and are now a ubiquitous tool in modern number theory as well as many other fields including algebraic topology, representation theory and algebraic geometry. The idea is to consider the completion of \mathbb{Q} with respect to the absolute value defined by $|x|_p = p^{-n}$ for non-zero $x \in \mathbb{Q}$ where $x = p^n \frac{a}{b}$ with $a, b, n \in \mathbb{Z}$ and a, b coprime to p . The resulting field \mathbb{Q}_p gives a neat way of packaging the information of congruences modulo p^n for all n and is the basic example of a local field. From this point of view, local fields are objects lying on the interface between algebra and analysis and the techniques used to study them involve an interesting mix of the two subjects.

This course will cover the basic theory of local fields, as well as some more advanced topics such as local class field theory, and is likely to be useful for students interested in studying other Part III courses on number theory. Topics likely to be covered include:

Absolute values on fields; valuations; Hensel's Lemma; structure of local fields;

Extensions of complete fields; Dedekind domains; Decomposition groups;

Ramification theory; higher ramification groups; Weil group;

Statements of local class field theory; formal groups; Lubin–Tate theory.

Prerequisites

Basic algebra up to and including Part II Galois theory as well as knowledge of concepts in point set topology and metric spaces are essential pre-requisites. It will be assumed that students have attended a first course in algebraic number fields.

Literature

1. J.W.S. Cassels, *Local fields*, Cambridge University Press, 1986. .
2. J.W.S. Cassels; A. Fröhlich, *Algebraic Number Theory*, Academic Press, 1967
3. J. Neukirch, *Algebraic Number Theory*, Springer–Verlag, 1999.
4. J. P. Serre, *Local fields*, Graduate Texts in Mathematics, 67. Springer–Verlag, 1979.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.