Local Fields (M16)

Professor T A Fisher

The theory of local fields was introduced by Hensel at the end of the 19th century as an alternative approach to algebraic number theory. The basic idea is to consider the completions of a number field K at all absolute values, not just the ones arising from the embeddings of K into the reals or complexes. One can then borrow techniques from analysis to study K and its finite extensions in a way that focuses on their behaviour at just one prime. For instance the analogue of the Newton-Raphson method for root finding goes by the name of Hensel's lemma.

Nowadays, local fields have established themselves as a natural tool in many areas of number theory and also in subjects like representation theory, algebraic topology and arithmetic geometry (e.g. elliptic curves).

The course will begin by introducing the field of p-adic numbers (where p is a prime). This is the completion of the field of rational numbers $\mathbb{Q}$ with respect to the p-adic absolute value defined for non-zero $x \in \mathbb{Q}$ by $|x|_p = 1/p^n$ where $x = p^n a/b$ with p not dividing a or b. The resulting field $\mathbb{Q}_p$ gives a convenient way of packaging information about congruences modulo p^n for all n, and is the basic example of a local field.

Topics to be covered include: absolute values on fields, valuations, complete fields and their extensions, inverse limits, Hensel's lemma and ramification theory. If time permits, then possible further topics include: local class field theory (statements only), the Hilbert norm residue symbol, and the Hasse-Minkowski theorem.

Prerequisites

Basic algebra up to and including Part II Galois theory as well as knowledge of concepts in point set topology and metric spaces are essential pre-requisites. It will be assumed that students have attended a first course in algebraic number fields.

Literature

1. J.W.S. Cassels, *Local fields*, Cambridge University Press, 1986.

2. J.W.S. Cassels, A. Fröhlich (eds.), *Algebraic number theory*, Academic Press, 1967.

3. G.J. Janusz, *Algebraic number fields*, AMS, 1996.

4. J. Neukirch, *Algebraic number theory*, Springer–Verlag, 1999.

5. J.P. Serre, *Local fields*, Graduate Texts in Mathematics, 67, Springer–Verlag, 1979.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.