Additive Prime Number Theory

Non-Examinable (Graduate Level)

Dr. L. Grimmelt

This course provides an introduction to the application of Fourier techniques in additive questions involving primes. It will cover both classical results like Vinogradov's sums of three primes theorem, as well as recent developments like Green's proof of Roth's theorem in the primes. This is an area that has a rich history, dating back to Hardy and Littlewood. Several of the introduced techniques, for example type I/II sums and the transference principle are still highly relevant for current research.

The course will be organised along the lines of

- Part I: Fourier series, convolutions, Roth's theorem
- Part II: The ternary Goldbach Problem, Vinogradov's contribution
- Part III: Roth's theorem in the primes, the fourier analytic transference principle

Prerequisites

This course assumes basic number theoretical knowledge (as for example covered in Part II number theory) and analysis (the Part IB course complex analysis is more than sufficient, and not strictly necessary).

The course is adjacent to both the courses on analytic number theory and additive combinatorics (in which Roth's theorem is also covered), but requires neither as prerequisite. It is ideal to accompany the course on analytic number theory.

Literature

- 1. Melvyn B. Nathanson, Additive Number Theory: The Classical Bases, Springer (1996)
- 2. B. Green, Roth's theorem in the primes, Annals of Mathematics, 161 (2005)
- 3. S. Prendiville, Four variants of the Fourier-analytic transference principle. Online Journal of Analytic Combinatorics (2007)

Additional Support

Two example sheets and opportunities to ask questions will be provided.