Information Theory (M16)

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Information theory is the mathematical foundation of the ideas and tools required for the quantitative description and analysis of the notion of information. The origin of the theory in Claude Shannon's landmark 1948 paper is motivated by questions in communications engineering, but since then information theory has forged deep connections not only with probability and statistics, but also with combinatorics and other areas of pure mathematics.

This will be an introductory course to the main mathematical aspects of information theory. We will discuss entropy as a measure of information, relative entropy as a natural distance between probability distributions, and mutual information as a dependence measure between random variables. Their properties will be established (monotonicity, chain rules, data processing inequalities, asymptotic equipartition) and different versions of Shannon's source coding theorems will be proved. Connections with large deviations theory and additive combinatorics will be developed. Finally, the Entropy Power Inequality (EPI) will be proved.

Pre-requisites

The only pre-requisite is knowledge of basic probability, although a certain level of maturity and familiarity with the use of probabilistic techniques will be helpful. Knowledge of advanced (including measure-theoretic) probability is not strictly necessary.

Literature

1. Mainstream introduction to information theory:
   T.M. Cover and J. Thomas. *Elements of Information Theory.* 2nd edition. Wiley-Interscience, 2006.

2. A more applied perspective:
   D. MacKay. *Information Theory, Inference, and Learning Algorithms.* Cambridge University Press, 2003. Available free online at: https://www.inference.org.uk/itprnn/book.pdf.

3. Theoretical connections with ergodic theory and probability...
   P. Billingsley. *Ergodic Theory and Information.* J. Wiley, 1965.

4. ... and: P. Shields. *The Ergodic Theory of Discrete Sample Paths.* American Mathematical Society, 1996.

5. Large deviations theory:
   A. Dembo and O.Zeitouni. *Large deviations techniques and applications.* Springer Science & Business Media, 2009.

6. Background on additive combinatorics:
   T. Tao and H. Van Vu *Additive combinatorics.* Cambridge University Press, 2006.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.