

Concentration Inequalities (L16)

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Informally speaking, the *concentration of measure phenomenon* asserts that if X_1, \dots, X_n are independent random variables, then $f(X_1, \dots, X_n)$ does not deviate much from its mean provided that $f(x_1, \dots, x_n)$ is not too sensitive in any of its coordinates x_i . The topic of study of this course is the set of the various tools used to quantify this phenomenon via *non-asymptotic inequalities*. These inequalities find applications in many domains including statistics, learning theory, discrete mathematics, random matrix theory, information theory, and high-dimensional geometry.

The course will cover a selection of topics including:

- Markov's inequality and Chernoff bound; inequalities for sub-Gaussian random variables; Hoeffding's inequality and related inequalities
- Efron-Stein inequality; log-Sobolev and Poincaré inequality; basic information-theoretic inequalities; the entropy method
- Talagrand's inequality; the transport method; Marton's transport cost inequalities

Applications of the above inequalities to random matrices, Boolean analysis and combinatorics will be considered throughout the course.

Prerequisites

The only pre-requisite is knowledge of basic probability, although a certain level of maturity and familiarity with the use of probabilistic techniques will be helpful. Knowledge of advanced (including measure-theoretic) probability is not necessary, but would be useful.

Literature

1. S. Boucheron, G. Lugosi and P. Massart, *Concentration Inequalities: A Non-Asymptotic Theory of Independence*, Oxford University Press, 2013.
2. R. van Handel, *Probability in High Dimensions*. Available at <https://web.math.princeton.edu/~rvan/APC550.pdf>.
3. R. Vershynin, *High Dimensional Probability*. Cambridge University Press, 2018, also available at <https://www.math.uci.edu/~rvershyn/papers/HDP-book/HDP-book.pdf>.
4. M. Ledoux, *The Concentration of Measure Phenomenon*, Mathematical Surveys and Monographs, 2001.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.