The Continuum Hypothesis (M16)

Professor Benedikt Löwe

[This course has been offered under the title "Forcing & the Continuum Hypothesis" in 2024–25 and 2025–26 with exactly the same content and in 2023–24 as a 24-lecture course with some additional content.]

The *Continuum Hypothesis* CH is the statement "every uncountable set of real numbers is in bijection with the set of all real numbers"; equivalently, $2^{\aleph_0} = \aleph_1$.

When he presented his list of twenty-three problems for the 20th century at the *International Congress of Mathematicians* in Paris in 1900, David Hilbert listed the the question whether the Continuum Hypothesis is true as the very first problem on his list. It turned out that this question cannot be solved on the basis of ZFC: in 1938, Kurt Gödel developed the *constructible universe* to show that CH cannot be disproved in ZFC; in 1963, Paul Cohen invented the *method of forcing* to show that CH cannot be proved in ZFC.

In this course we shall study models of set theory in general, develop the methods of inner models and forcing, and see the two mentioned meta-mathematical proofs that resolve Hilbert's first problem. The two methods of inner models and forcing are the two most important construction methods for models of set theory and useful well beyond the immediate application of resolving Hilbert's first problem.

Prerequisites

The Part II course *Logic & Set Theory* or an equivalent course is essential. It is strongly recommended (but not strictly necessary) to take Part III *Model Theory* to supplement the discussion of models of set theory in this course.

Literature

Kenneth Kunen. *Set Theory. An Introduction to Independence Proofs.* Elsevier 1980 [Studies in Logic and the Foundations of Mathematics, Vol. 102].

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.