Model Theory (M16)

Dr Mervyn Tong

Model theory is a branch of mathematical logic in which a mathematical structure is studied in terms of its (first-order) theory, that is, the set of (first-order) sentences of which it is a model. Classically, this was driven by the desire to understand how much a mathematical structure is characterised by its theory, leading to deep classification results for the class of models of a given theory. More recently, model theorists have observed that many properties of mathematical structures arising naturally in fields such as geometry, number theory, and combinatorics are closely connected to the model-theoretic properties of their theories, and can thus be analysed by model-theoretic techniques. This course introduces some basic model-theoretic tools and ideas up to initial notions in stability theory.

We will cover a selection of the following topics.

- Preliminaries: substructures, homomorphisms, elementary equivalence, etc.
- Complete theories: Vaught's test and applications, such as the Ax–Grothendieck theorem.
- Ultraproduct structures and Łoś's theorem.
- Quantifier elimination: tests and applications.
- Types: type spaces, homogeneity, saturation, etc.
- The omitting types theorem and the Ryll-Nardzewski theorem.
- An introduction to stability theory, including various characterisations of stability.

Prerequisites

The Part II course *Logic and Set Theory*, or an equivalent course, is essential. We will also use standard notions from point-set topology.

Literature

1. David Marker. *Model Theory: An Introduction*, volume 217 of *Graduate Texts in Mathematics*. Springer, New York, 2002.
2. Katrin Tent and Martin Ziegler. *A Course in Model Theory*, volume 40 of *Lecture Notes in Logic*. Cambridge University Press, New York, 2012.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a 1.5-hour revision class in the Easter Term.