Model Theory (L16)

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Model theory is a branch of mathematical logic. Initially, the focus was on how far a set of sentences in first-order language determines the class of structures it describes (e.g. are two models of the same cardinality of a particular set of sentences isomorphic). Later, the subject evolved in several directions. For example, part of the attention shifted to contexts where the subsets of a structure that are definable are also mathematically meaningful (for instance, the definable sets in an algebraically closed field are the constructible sets). This is one of several ways in which model theory interacts with other areas of mathematics. Connections have emerged to real and algebraic geometry, number theory and, more recently, combinatorics. This course introduces some basic model-theoretic tools and ideas up to initial notions in stability theory.

We will cover a selection of the following topics:

- Preliminaries: structures, theories, elementarity (including elementary substructures, Tarski-Vaught test, downward Löwenheim-Skolem theorem).

- Examples of structures (relational structures, basic algebraic structures).

- Back and forth and Quantifier Elimination. Types, type spaces.

- Ultraproducts, Ultrapowers and Łoś's theorem.

- The compactness theorem for theories and for types.

- Prime models. Omitting types theorem, possibly including ω-categorical theories.

- Strongly Minimal Structures.

- Stability theory.

- O-minimal structures.

Prerequisites

The Part II course *Set Theory and Logic*, or an equivalent course, is essential.

Literature

1. D. Marker *Model Theory: An Introduction* Vol. 217. Springer Science & Business Media, 2006.

2. Katrin Tent and Martin Ziegler, *A Course in Model Theory* Lecture Notes in Logic 40. Cambridge University Press, 2012.

3. W. Hodges, *A shorter model theory.* Cambridge University Press, Cambridge, 1997.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.