Symplectic Topology (L16)

Dr A. Ward

This course is intended as a first introduction to symplectic topology, the study of smooth manifolds equipped with a closed and non-degenerate differential 2-form. This class of spaces generalizes the phase spaces that appear in the study of classical mechanics; other instances of symplectic manifolds include real surfaces equipped with a volume form, complex algebraic varieties, and co-adjoint orbits of Lie groups. As this collection of examples might suggest, symplectic topology has important connections with many other fields of study such as algebraic geometry and low-dimensional topology. It is also interesting in its own right, and is an active area of mathematical research today.

The topics covered in this course will include:

- Symplectic linear algebra. Symplectic manifolds, symplectic/Lagrangian submanifolds. Almost complex structures and Chern classes.
- Symplectomorphisms, Hamiltonian dynamics, the flux homomorphism.
- Moser's trick, Darboux's theorem, Weinstein's neighborhood theorem.
- Constructions of symplectic manifolds including symplectic blow-ups, Lefschetz pencils, symplectic reductions.
- Introduction to pseudo-holomorphic curves. Gromov's non-squeezing theorem.

Pre-requisites

Some familiarity with complex analysis, vector bundles on smooth manifolds, and de Rham cohomology will be essential. Differential geometry and algebraic topology from Part III (or the equivalent) are strongly recommended.

Literature

1. Cannas da Silva, A. *Lectures on Symplectic Geometry.* Springer, 2001.
2. McDuff, D and Salamon, D. *Introduction to Symplectic Topology (3rd Edition).* Oxford Univ. Press, 2017.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a revision class in the Easter Term.