

Simplicial Homotopy Theory (L24)

Professor J. Hahn

Simplicial sets simultaneously serve as combinatorial models for topological spaces up to homotopy equivalence (through the theory of Kan complexes) and as the foundational objects in the quasi-categorical approach to higher category theory. In this course we develop the basics of homotopy theory using the language of Kan complexes. We then introduce infinity categories, defining and exploring the infinity category of Kan complexes. Topics will include:

- Homotopy groups
- Fibrations and cofibrations
- The Whitehead and Hurewicz theorems
- Postnikov towers
- Quasi-categories, including the ∞ -categories of Kan complexes and chain complexes
- Homotopy (co)limits
- The Serre spectral sequence
- Brown representability.

Prerequisites

While there will be some review, the course assumes a working knowledge of both 1-category theory (colimits, limits, adjunctions, and the Yoneda embedding) and the singular homology of topological spaces.

Literature

The readings below may prove useful, but in total cover far more than the course content:

1. Lurie, J. *Kerodon*. Available at <https://kerodon.net/>.
2. Goerss, P. and Jardine, R. *Simplicial Homotopy theory*. Birkhäuser, 2009.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.