Knots and Knot Concordances (L25)

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Knots are embedded circles in S^3. In a 3-dimensional sense, knot theory is a study of their ambient isotopy classes in S^3. However, by considering S^3 as the boundary of the 4-ball B^4 and studying the embedded surfaces bounded by knots, one gets a much more subtle information. In this course, we take this 4-dimensional approach and explore the theory of knot concordances.

The course will cover a selection of topics including:

- Knots and elementary knot operations
- Fox-Milnor condition
- Levine-Tristram signatures
- The Knot concordance group
- The algebraic concordance group
- Slice genus of knots
- Double slicing of knots

Prerequisites

The courses assumes a working knowledge of Algebraic Topology (in particular, fundamental groups, homology, and cohomology).

Literature

1. D. Rolfsen, *Knots and Links.* American Mathematical Society, 2023.
2. R. E. Gompf and A. I. Stipsicz, *4-manifolds and Kirby calculs.* Vol. 20. American Mathematical Society, 2023.
3. P. Teichner, *Slice Knots: Knot Theory in the 4th Dimension.* Unpublished lecture notes by Julia Collins and Mark Powell. Available online at https://people.mpim-bonn.mpg.de/teichner/Math/ewExternalFiles/Lecture-notes-sliceknots.pdf.
4. A. Conway, *The Levine-Tristram signature: a survey.* 2019-20 MATRIX Annals. Cham: Springer International Publishing, 2021. 31-56.
5. J. Levine, *Knot cobordism groups in codimension two.* Commentarii Mathematici Helvetici 44.1, 1969. 229-244.
6. D. Cimasoni and V. Florens, *Generalized Seifert surfaces and signatures of colored links.* Transactions of the American Mathematical Society 360.3 (2008): 1223-1264.
7. M. Powell, *The four genus of a link, Levine-Tristram signatures and satellites.* Journal of Knot Theory and Its Ramifications 26.02 (2017): 1740008.
8. P. Orson and M. Powell, *A lower bound for the doubly slice genus from signatures.* New York Journal of Mathematics 27 (2021): 379-392.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.