# Differential Geometry (M24)

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Differential Geometry provides a general framework for doing *calculus on spaces*. It provides one important perspective to modern geometry, by emphasizing on the local geometric structures defined via linear algebra and derivatives (such as the notion of Riemannian metric, curvature, etc.). This is complementary to Topology, which emphasizes on global qualitative information, and Algebraic Geometry, which studies the spaces through the algebraic functions on them. Differential Geometry also draws much of its motivations from Theoretical Physics, such as Einstein metrics, minimal surfaces etc., and some of its ideas have influenced the development of Analysis. A tentative syllabus is as follows:

- Differentiable manifolds. Definition and examples of manifolds, tangent and cotangent spaces. Submanifold, embedding and immersions. Differential forms and Stokes formula. Cartan's magic formula. De Rham cohomology.
- Riemannian metric. Geodesics, Levi-Civita connection, covariant derivative. Riemannian curvature, Ricci and scalar curvature, Laplacian, vaccum Einstein equation. Hodge decomposition and harmonic representative.
- Minimal surfaces. Second fundamental form, mean curvature, area minimisation, stability, monotonicity formula. Time permitting, we may discuss some applications of minimal surface theory.

### Prerequisites

An essential prerequisite is a working knowledge of linear algebra and multivariate calculus. Previous exposure to curves and surfaces in $R^3$ would be useful. Physics background is not required but may be useful motivations.

### Literature

Some of the previous year lecture notes will have significant overlaps with this course, and would be useful supplementary material.

- A. Kovalev: https://www.dpmms.cam.ac.uk/ agk22/teaching.html
- I. Smith (unofficial notes): https://minterscompactness.wordpress.com/lecture-notes/

Some standard textbooks include:

- F.W. Warner, Foundations of differentiable manifolds and Lie groups, Springer-Verlag, 1983.
- M.P. do Carmo, Riemannian geometry. Birkhäuser, 1993.

### Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term. Supplementary notes may be provided.