Differential Geometry (M24)

Dr A. Kovalev

This course is intended as an introduction to modern differential geometry. It can be taken with a view of further studies in Geometry and Topology and should also be suitable as a supplementary course if your main interests are e.g. in Analysis or Mathematical Physics. A tentative syllabus is as follows.

- *Local Analysis and Differential Manifolds.* Definition and examples of manifolds, matrix Lie groups. Tangent vectors, the tangent and cotangent bundle. Geometric consequences of the implicit function theorem, submanifolds. Exterior algebra of differential forms. Orientability of manifolds. Partition of unity and integration on manifolds, Stokes' Theorem. De Rham cohomology.
- *Vector Bundles.* Structure group, principal bundles. The example of Hopf bundle. Bundle morphisms. Three views on connections: vertical and horizontal subspaces, Christoffel symbols, covariant derivative. The curvature form and second Bianchi identity.
- *Riemannian Geometry.* Connections on manifolds, torsion. Riemannian metrics, the Levi–Civita connection. Geodesics, the exponential map, Gauss' Lemma. Decomposition of the curvature of a Riemannian manifold, Ricci and scalar curvature, low-dimensional examples. The Hodge star and Laplace–Beltrami operator. Statement of the Hodge decomposition theorem (with a sketch-proof, time permitting).

Prerequisites

An essential prerequisite is a working knowledge of linear algebra (including dual vector spaces and bilinear forms) and of multivariate calculus (e.g. differentiation in several variables and the inverse function theorem). Exposure to some ideas of classical differential geometry (of curves and surfaces in $\mathbf{R}^3$) would be useful.

Literature

[1] R.W.R. Darling, *Differential forms and connections.* CUP, 1994.

[2] S. Gallot, D. Hulin, J. Lafontaine, *Riemannian geometry.* Springer-Verlag, 1990.

[3] V. Guillemin, A. Pollack, *Differential topology.* Prentice-Hall Inc., 1974.

[4] F.W. Warner, *Foundations of differentiable manifolds and Lie groups*, Springer, 1983.

Roughly, half of the course material is found in [4]. The book [3] covers the required topology. On the other hand, [1] which has a chapter on vector bundles and on connections assumes no knowledge of topology. Both [1] and [2] have a lot of worked examples. There are many other good differential geometry texts.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term. Printed notes will be available from
https://www.dpmms.cam.ac.uk/~agk22/teaching.html