Algebraic Topology (M24)

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Algebraic Topology studies topological spaces by associating to them algebraic invariants, primarily abelian groups and commutative rings. It permeates modern pure mathematics and theoretical physics. This course will focus on (co)homology, with an emphasis on applications to the topology of manifolds. We will cover:

- singular homology and cohomology and its main calculational devices (eg. Mayer–Vietoris),
- cup-products in cohomology,
- the Künneth theorem,
- vector bundles and the Thom isomorphism theorem,
- the cohomology of manifolds and Poincaré duality.

The course will not assume prior knowledge of algebraic topology, but will start off rather fast in order to reach the more interesting material, so some previous exposure to chain complexes and simplicial homology would definitely be helpful.

Prerequisites

Basic topology: topological spaces, compactness and connectedness, at the level of Sutherland's book. Some knowledge of the fundamental group would be helpful though not a requirement. The book by Hatcher is especially recommended, but there are many other suitable texts and many online resources. The Part III Differential Geometry course will also contain useful, relevant material.

Literature

1. Hatcher, A. *Algebraic Topology.* Cambridge Univ. Press, 2002.
2. May, P. *A concise course in algebraic topology.* Univ. of Chicago Press, 1999.
3. Sutherland, W. *Introduction to metric and topological spaces.* Oxford Univ. Press, 1999.

Additional support

The course will emphasise examples and computations; it will be accompanied by four question sheets with associated Examples Classes, which will again involve applying the general theory to do explicit calculations and solve geometric problems. There will be a one-hour revision class in the Easter term.