Slow Viscous Flow (M24)

Prof. J.R. Lister

In many flows of natural interest or technological importance, the inertia of the fluid is negligible. This may be due to the small scale of the motion, as in the swimming of micro-organisms and the settling of fine sediments, or due to the high viscosity of the fluid, as in the processing of glass and the convection of the Earth's mantle.

The course will begin by presenting the fundamental principles governing flows of negligible inertia. A number of elegant results and representations of general solutions will be derived for such flows. The motion of rigid particles in a viscous fluid will then be discussed. Many important phenomena arise from the deformation of free boundaries between immiscible liquids under applied or surface-tension forcing. The flows generated by variations in surface tension due to a temperature gradient or contamination by surfactants will be analysed in the context of the translation and deformation of drops and bubbles and in the context of thin films. The small cross-stream lengthscale of thin films renders their inertia negligible and allows them to be analysed by lubrication or extensional-flow approximations. Problems such as the fall of a thread of honey from a spoon and the subsequent spread of the pool of honey will be analysed in this way. Inertia is also negligible in flows through porous media, such as the extraction of oil from sandstone reservoirs, movement of groundwater through soil or the migration of melt through a partially molten mush. Some basic flows in porous media may be discussed.

The course aims to examine a broad range of slow viscous flows and the mathematical methods used to analyse them. The course is thus generally suitable for students of fluid mechanics, and provides background for applied research in geological, biological or rheological fluid mechanics.

Prerequisites

As described above in the introduction to courses in Continuum Mechanics. Familiarity with basic vector calculus including Cartesian tensors and the Einstein summation convention is particularly useful for the first half of the course.

Preliminary Reading

1. D.J. Acheson. *Elementary Fluid Dynamics.* OUP, 1990. Chapter 7
2. G.K. Batchelor. *An Introduction to Fluid Dynamics.* CUP, 1970. pp.216–255.
3. L.G. Leal. *Laminar flow and convective transport processes.* Butterworth, 1992. Chapters 4 & 5.

Literature

1. J. Happel & H. Brenner. *Low Reynolds Number Hydrodynamics.* Kluwer, 1965.
2. S. Kim & J. Karrila. *Microhydrodynamics: Principles and Selected Applications.* 1993.

Additional support

Four two-hour examples classes will be given by the lecturer to cover the four examples sheets. There will be a further revision class in the Easter Term.