Probabilistic Combinatorics (M24)

Julian Sahasrabudhe

This course will introduce a few of of the fundamental methods in probabilistic combinatorics. In this course I aim to cover many of the following topics.

Basics of set systems Sperner's theorem; intersecting families of sets; isoparametric inequalities in the hypercube.

Probabilistic methods First moment method; deletion method; Lovász local lemma; upper and lower bounds for the Ramsey numbers $R(3,k)$; semi-random method.

Dependent random choice
Extremal numbers of bipartite graphs; Ramsey-Turán; Sidorenko conjecture; Ramsey numbers of hypercubes.

Pseudo-random graphs
Size Ramsey number for a path; Alon-Rödl bound on multi-colour Ramsey numbers; Graham-Rödl-Ruciński on Ramsey numbers of bounded degree graphs.

Szemerédi's regularity lemma
Basic statement and philosophy; Erdős-Stone, triangle removal lemma; Roth's theorem; the number of triangle-free graphs; Ramsey-Turán.

Graph containers
Kleitman-Winston containers theorem; the number of C_4-free graphs; lower bound on $R(4,k)$.

Prerequisites

Familiarity with the topics and flavor of Part II graph theory (or an equivalent course) is useful and recommended although not strictly required.

Literature

This course draws from various sources, and there is no one good resource for course. However, many of the topics are fairly standard and one can find lecture notes online. The first few lectures will have quite a bit of overlap with Bollobás's book *Combinatorics*. Some of the main topics in the middle part of the course are covered in in R. Morris and R.I. Oliveira's lecture notes. Alon and Spencer's *The probabilistic method* is also a good resource.

1. N. Alon and J. Spencer *The Probabilistic Method*, Wiley (any edition).
2. B. Bollobas *Set Systems, Hypergraphs, Families of Vectors and Combinatorial Probability*, Cambridge University Press.
3. R. Morris and R.I. Oliveira *Extremal and Probabilistic Combinatorics*, available at https://impa.br/wp-content/uploads/2017/04/28CBM_04.pdf. 1993.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.