Ramsey Theory (M16)

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Ramsey theory is concerned with the general question of whether, in a large amount of disorder, one can find regions of order. Alternatively, given a certain structure, if one 'breaks' up 'the universe' into many many pieces, can we find one piece that contains that structure? A typical example is van der Waerden's theorem, which states that whenever we partition the natural numbers into finitely many classes there is a class that contains arbitrarily long arithmetic progressions. So the universe is the natural numbers and the structure is the arithmetic progression.

This is a purely combinatorial course. Ramsey theory is remarkably attractive: we study questions that are very natural and easy to appreciate, but whose answers rely on a great variety of beautiful methods. Not only that, but almost no two questions can be solved in the same way – at least one brand new idea is needed.

We shall cover a number of 'classical' Ramsey theorems, such as Gallai's theorem and the Hales-Jewett theorem, as well as some more recent developments. There will also be several indications of open problems. We hope to cover the following material.

- Monochromatic Systems.
  Ramsey's theorem (finite and infinite). Canonical Ramsey theorems. Colourings of the natural numbers. Focusing and van der Waerden's theorem. Combinatorial lines and the Hales-Jewett theorem. Applications, including Gallai's theorem.
- Partition Regular Equations.
  Definitions and examples. The columns property and Rado's theorem. Applications. (m, p, c)-sets and Deuber's theorem. Ultrafilters. the Stone-Čech compactification. Idempotent ultrafilters and Hindman's theorem.
- Euclidean Ramsey Theory.
  Basic definitions. Ramsey sets and spherical sets. Triangles are Ramsey. Regular polygons are Ramsey. Transitive sets. The block sets conjecture.

Prerequisites

There are almost no prerequisites – the course will start with a review of Ramsey's theorem, so even prior knowledge of this is not essential. At various places we shall make use of some very basic concepts from topology, such as metric spaces and compactness – however, nothing more than basic knowledge any mathematics undergraduate, regardless of personal interest, should posses.

Literature

1. R. Graham, B. Rothschild and J. Spencer, *Ramsey Theory.* John Wiley, 1990.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.