Ramsey theory on graphs (M16)

Dr J. Sahasrabudhe

This course will introduce several of the fundamental methods in modern combinatorics by focusing on the Ramsey numbers $R(\ell, k)$ and related objects. In this course I aim to cover most of the following topics:

- Upper bounds for classical ramsey numbers Erdős-Szekeres upper bound on $R(\ell, k)$; The Ajtai-Komlos-Szemeredi theorem when ℓ is bounded; recent progress on R(k).
- Lower bounds for classical ramsey numbers Erdős' lower bound on R(k); The Lovász Local lemma and R(3, k); The method of Graph containers and the Alon-Rődl technique for multicolour Ramsey numbers; Recent lower bounds on R(4, k).
- Hypergraph ramsey numbers Erdős-Hajnal upper bounds for diagonal hypergraph ramsey; the "stepping up" method.
- Szemerédi's regularity lemma Basic statement and philosophy; triangle removal lemma; Roth's theorem; Ramsey numbers of bounded degree graphs.
- Dependent random choice Extremal numbers of bipartite graphs; Ramsey-Turán; Ramsey numbers of hypercubes.
- Related colouring topics Graphs with high girth and chromatic number; hypergraphs with property B.

Prerequisites

Familiarity with the topics and flavor of Part II graph theory (or an equivalent course) is useful and recommended although not strictly required.

Literature

This course draws from various sources, and there is no one good resource for course. However, many of the topics are standard and one can find lecture notes online. The first few lectures will see quite a bit of overlap with David Conlon's course on the same topic. Then the latter part of the course is also covered in R. Morris and R.I. Oliveira's lecture notes. The The probabilistic method is a good source for more probabilistic examples we are using.

- 1. N. Alon and J. Spenser *The Probabilistic Method*, Wiley (any edition).
- 2. D. Conlon *Graph Ramsey Theory*, Lecture notes available at http://www.its.caltech.edu/~dconlon/Ramsey-course.html.
- 3. R. Morris and R.I. Oliveira *Extremal and Probabilistic Combinatorics*, available at https://impa.br/wp-content/uploads/2017/04/28CBM_04.pdf. 1993.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.