Introduction to Additive Combinatorics (M16)

Professor J. Wolf

This course provides an introduction to additive combinatorics, a fast-moving area of mathematics motivated by long-standing questions about additive structure in sets of integers. It will attempt to reflect both the elementary appeal of the key problems in the area as well as the breadth of techniques (analytic, combinatorial, algebraic, probabilistic) used to tackle them. By the end of the course, students should be able to appreciate various recent breakthroughs in the area.

The course will be organised along the following lines.

- Part I: Combinatorial methods
  Plünnecke's inequality for sum sets; the Freiman-Ruzsa theorem
- Part II: Fourier-analytic techniques
  Bogolyubov's lemma; Roth's theorem on 3-term arithmetic progressions
- Part III: Probabilistic methods
  the Balog-Szemerédi-Gowers theorem; Croot-Sisask almost periodicity and applications
- Part IV: Further topics (if time permits)
  algebraic constructions and obstructions; beyond 3-term progressions

Prerequisites

This course assumes basic familiarity with finite abelian groups, linear algebra, discrete probability and graph theory, at a level found in most undergraduate mathematics degrees. As such, it should be accessible to a broad range of students.

Literature

The following books give a flavour of the course, but should not be regarded as compulsory or definitive reading.

1. T. Tao and V. Vu. *Additive Combinatorics* (Cambridge Studies in Advanced Mathematics). Cambridge University Press, 2006.
2. Y. Zhao. *Graph Theory and Additive Combinatorics: Exploring Structure and Randomness*. Cambridge University Press, 2023.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.